CHARACTERISTICS AND INTERRELATIONS OF FORMS

ARCH 115
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Mathematics in Architecture
Mathematics can, broadly speaking, be subdivided into the study of quantity - arithmetic, structure - algebra, space - geometry and change - analysis.

In addition to these main concerns, there are also sub-divisions dedicated to exploring links from the heart of mathematics to other fields:

to logic,
to set theory (foundations)
to the empirical mathematics of the various sciences
to the rigorous study of uncertainty
Lecture Topics

. Mathematics and Architecture are wholly related with each other.

. All architectural products from the door to the building cover a place in space. So, all architectural elements have a volume. One of the study field of mathematics, name is geometry is totally concern this kind of various dimensional forms such as two or three or four. If you know geometry well, you can create true shapes and forms for architecture. For this reason, in this course, first we’ll look to the field of geometry especially two and three dimensions.

. Our second goal will be learning construction (drawing) of this two or three dimensional forms.
• Architects intentionally or accidently use ratio and mathematical proportions to shape buildings. So, our third goal will be learning some mathematical concepts like **ratio and proportions**.

• In this course, we explore the many places where the fields of art, architecture and mathematics. For this aim, we’ll try to learn concepts related to these fields like **tetractys, magic squares, fractals, penrose tilling, cosmic rythms** etc.

• We will expose to you to wide range of art covering a long historical period and great variety of styles from **the Egyptians** up to **contemporary world** of art and architecture.
Geometry
Geometry and Art & Architecture

• Geometry is the study of space and architecture in the broadest sense of the word, is the creation of space by construction and subdivision.

• The two disciplines are virtually inseperable with one distinction. Geometry can exist without architecture but architecture can not exist without geometry. Also, geometry is not all of architecture but is an essential part of it.

• Not only architecture is related with geometry, but also painting and sculpture are related.
Although closely related, geometry of architecture is different from the geometry of painting and sculpture;

“Architectural space must serve the needs of humans” with some exactness: floor must be level, stairs must be straight, the law of gravity must be respected, builds must be buildable. However, you are free, while you are creating space in painting.

For example, marvelous fantasy geometry of M. C. Escher (1898-1972 Dutch painter) transcends the world of the practicing architect.
“Architectural surfaces be built up through the repetition of identical pieces and process”, such as the laying up of a brick wall. If every piece is different, both the manufacturing process and the assembly process are impractical. However, you can create whatever you want in painting and sculpture. You are not in the restrictions of mass production.

For example, the highly geometric, colorful, and volatile paintings of Victor Vasarely (1908-1997 French painter) violate an essential requirement of architectural geometry.
In the recent past, art and architecture did come together in geometry. The paintings of Mondrian (1872-1912 Dutch painter) and architecture of Mies van Der Rohe (1886-1969 German architect) both relied on refined proportions of rectangles and squares.

To an architect, of course, the rectangle is a basic tool, but it is rare that an artist with the fluidity of paint at his command remains enchanted with the right-angle discipline as did Mondrian.

Fransworth House, 1951, Illinois
In architecture, all constructed works be perfectly in tune with nature.

The relationship of building to nature is first all defensive. A building must be designed to resist the onslaughts of nature – rain, wind, fire, snow, earthquake, temperature extremes and so on.

These primary purposes of shelter and protection dictate specific materials and geometries depending on the resources available. Roofs are sloped or domed, walls are braced at right angles, openings are framed and arched and so on.
As well as provide shelter, buildings may encompass the landscape because of its "life" characteristics. It breaths through its ventilation system, has power in its veins, a body temperature, a circulation system, possibly a hearth, and a skeleton covered and protected by a skin. All living things adapt to their surroundings. Building can blend into the surroundings with a minimum disruption. They can conserve natural resources.

An example; Completely harmonies integration of rectangular shapes into a natural setting is Falling Water, a dwelling designed by Frank Lloyd Wright in 1936. The major task is to find the harmonies of the urban environment.
Geometry is not only the subject of architects and artists, but also the subjects of designers, planners, engineers.

. Geometry in **interior design**; are from the covering material to the furniture.

. Geometry in **structural design**; are all of civil engineering products from a building to the bridges.

. Geometry in **city planning**; from the ancient city planning to the contemporary city planning.

. Geometry in **landscape architecture**; from the landscape objects to the landscape planning.
The Philosophy of Geometry and Point of Departure Concept

• In the most general sense, geometry is a branch of mathematics studying the space and the patterns and objects which can be designed in the space.

• If there is no philosophy then there is no science.

• Yes, geometry did have a philosophy.

• Every form, whether living or non-living, in the nature - including human beings - was based on a geometric foundation. Every being or object which occupies a place in the space and which has a volume has also a geometry.

• This, the foundation of geometry, is the point which is the symbol of existence and unity.

• The big bang theory, which tried to prove the creation of the universe and suggested that the universe started from a point and continued to expand, told the same thing, existence and unity of God and every being carrying a part of it was the same thing.
• The point as a symbol for unity and source.

• The first move, creating a line.

• The second move, the arc creating a boundary.

• The closing of the circle to form a domain.
• The science of geometry dates back to old times and its foundations were laid by the Old Egyptians and Greeks.

• On the other hand, an Anatolian philosopher Thales of Miletus (624-546 BC) is referred to as the pioneer of philosophy and science. Thus, the most beautiful examples to use of geometry were practiced in Anatolia until the century before the awakening of the Western Europe.
The Seljuks, which provided the most beautiful examples of geometric embellishment created masterpieces in Anatolia. It is still a matter of discussion how they were masters of these forms at that time.
• The golden ratio discovered by the Old Egyptians and Greeks is geometric and numeric proportion correlation assumed to provide the most perfect sizes in terms of harmony, observed between the parts of a whole in mathematics and art.

• It has been generally used in architecture and art. It is widely believed that buildings constructed based on this proportion are aesthetic.
• Leonardo da Vinci even tried to prove that the geometric form of the human being is a result of this ratio. Similarly, the pi number, which was actually derived from the golden ratio, importance of which was well known by the old mystics and which constitutes the foundation of the universe and our physical and biological world is a geometric form rather than an arithmetic proposition.
• The foundation of the universe and living creatures is based on this form. While the pentagon is the simplest form to explain the golden ratio, the golden ratio has its examples in the nature with the golden spiral drawn from the golden quadrilateral.

• Yes, everything began with a point, the point became the center of the circle, the circle defined the lines, the lines created surfaces, the surfaces turned into volumes and geometries of all living and non-living creatures were created.
Classification of Geometrical Shapes for Architectural Design
Geometry is all about shapes and their properties.

- Geometry can be divided into two main groups;
  - **Plane Geometry**, is about flat shapes like line and all kind of surfaces. Shapes that can be drawn in a paper in plane geometry.
  
  - **Solid Geometry**, is about three dimensional objects like all kind of space covering objects.
In Geometry, we can make classification of geometrical shapes according to their dimensions for architectural design.

In this, there are six main groups:

- **0 Dimensional Forms with no surface** (Point)
- **1 Dimensional Forms with 1 Dimensional Surface** (Line, Angle)
- **2 Dimensional Forms with 1 Dimensional Surface** (Circle, Ellipse, Parabola, Hyperbola, Polygons)
- **3 Dimensional Forms with 1 Dimensional Surface** (Helix, Hemihelix)
- **3 Dimensional Forms with 2 Dimensional Surfaces** (Platonic Solids, Quadrics)
- **4 Dimensional Forms with 3 Dimensional Surfaces** (Polychrons)
# Common Geometrical Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Example</th>
<th>In Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>△</td>
<td>Triangle</td>
<td>ΔABC has 3 equal sides</td>
<td>Triangle ABC has three equal sides</td>
</tr>
<tr>
<td>∠</td>
<td>Angle</td>
<td>∠ABC is 45°</td>
<td>The angle formed by ABC is 45 degrees.</td>
</tr>
<tr>
<td>⊥</td>
<td>Perpendicular</td>
<td>AB⊥CD</td>
<td>The line AB is perpendicular to line CD</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Parallel</td>
</tr>
<tr>
<td>°</td>
<td>Degrees</td>
<td>360° makes a full circle</td>
<td></td>
</tr>
<tr>
<td>ℓ</td>
<td>Right Angle (90°)</td>
<td>ℓ is 90°</td>
<td>A right angle is 90 degrees</td>
</tr>
<tr>
<td>\overline{AB}</td>
<td>Line Segment &quot;AB&quot;</td>
<td>\overline{AB}</td>
<td>The line between A and B</td>
</tr>
<tr>
<td>\overrightarrow{AB}</td>
<td>Line &quot;AB&quot;</td>
<td>\overrightarrow{AB}</td>
<td>The infinite line that includes A and B</td>
</tr>
<tr>
<td>\overrightarrow{AB}</td>
<td>Ray &quot;AB&quot;</td>
<td>\overrightarrow{AB}</td>
<td>The line that starts at A, goes through B and continues on</td>
</tr>
<tr>
<td>≅</td>
<td>Congruent (same shape and size)</td>
<td>ΔABC ≅ ΔDEF</td>
<td>Triangle ABC is congruent to triangle DEF</td>
</tr>
<tr>
<td>~</td>
<td>Similar (same shape, different size)</td>
<td>ΔDEF~ΔMNO</td>
<td>Triangle DEF is similar to triangle MNO</td>
</tr>
<tr>
<td>\therefore</td>
<td>Therefore</td>
<td>a=b \therefore b=a</td>
<td>a equals b, therefore b equals a</td>
</tr>
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</table>
1 Dimensional Forms with 1Dimensional Surface (Line and Angles)
In geometry, **LINE** is straight (no curves), has no thickness, and extend in both directions without end. (infinetly)

**Transversal** is a line that a crosses at least two other lines. **Parallel** are two or more lines are not intersect each other.
CONSTRUCTION OF LINES

• And now, we will look one-dimensional forms with 1 dimensional surface constructions in geometry related with the line without using protractor, by the help of compass and straightedge.

  a line bisector
  a perpendicular to a point on a line
  a perpendicular to a point NOT on a line
  a parallel line through a point
  cut a line into the segments
. Place the compass at one end of line

. Adjust the compass to slightly longer than half the line length.
. Draw arcs above and below the line.
. Keeping the same compass width, draw arcs from other end of line.

. Place ruler where the arcs cross, and draw the line.

Construction of Line Bisector
. Draw a circle from the point on a line.

. Adjust the compass slightly bigger half of the arch.
. Draw new arcs from intersections of line and arcs.

. Place ruler from line to the intersection of arcs to draw perpendicular to a point on a line.

Construction of Perpendicular to a Point on a Line
Construction of Perpendicular to a Point NOT on a Line
. Draw a line from the starting point of line through point.
. Draw an arc from the starting point of line to cut two lines and draw same arc from the point.

Place compass to the intersection of first arc and drawn lines,
. Draw a new arc from this point to the intersection of the arc and line.
. Draw same size arc from the other point on the drawn line.

. Place ruler from the point to this intersection of these arcs to draw parallel line through a point.

Construction of Parallel Line Through a Point
1. Draw a line from the start point, heading somewhat upwards.

2. Use the compass to divide it into 3 segments.

3. Use the compass to create a parallel line heading backwards and downwards from the endpoint.
4. Use the compass to divide it into 3 segments.
5. Connect the intersection points of the two new lines, and where they cross the original line it will be subdivided.

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Construction of Cutting a Line into N Segments
There are six main types of angles measuring with degree (°), drawing by the help of protractor:

1. **Acute angles**: is the small angle is less than 90°

2. **Right angles**: is an interval angle which is 90°

3. **Obtuse angles**: is more than 90° but less than 180°.
4. **Straight angles**; is 180°, changes the direction to opposite way.

5. **Reflex angles**; is more than 180° but less than 360°.

6. **Full rotation angles**; is 360°. (Probably because old calendars such as the Persian Calendar used 360 days for a year - when they watched the stars they saw them revolve around the North Star one degree per day. Also 360 can be divided evenly by 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120 and 180, which makes a lot of basic geometry easier.)
For the angles there are two concepts;

**Congruent Angles**; have the same angle in degree or radians.

**Vertically opposite angles**; are the angles opposite each other when two lines accross. Vertical angles is always equal angles.
There are two major groups of angle according to their placement.

1. According to their sum;

   **Supplementary angles;** are two angles add up the 180°.

   ![Image of supplementary angles]

   **Complementary angles;** are two angles add up the 90°.

   ![Image of complementary angles]
2. According to their place on a transversal:

**Corresponding Angles;** the angles in matching corners.

**Consecutive angles;** the pairs of angle one side of transversal but inside.

**Alternate interior angles;** The pairs of angles on opposite sides of the transversal but inside.

**Alternate exterior angles;** The pairs of angles on opposite sides of the transversal but outside.
CONSTRUCTION OF ANGLES

- And now, we will look one-dimensional forms with 1 dimensional surface constructions in geometry related with angle without using protractor, by the help of compass and straightedge.

  an angle bisector
  a 30 degree angle
  a 45 degree angle
  a 60 degree angle
  a same (congruent) angle
1. Place the compass to the vertex of the angle.

2. Draw arcs which cut two arms of the angle.
   Draw new arcs slightly longer than half of the distance, with the center of points on the arms of the angle.

3. Place ruler where the arcs cross and vertex of the angle to draw bisector of the angle.

Construction of a Angle Bisector
Construction of a 60 Degree Angle

1. Place compass on starting point of line and draw an arc.

2. Place compass to the intersection point of line and arc. Adjust compass from this point with to reach start point.

3. Draw a new arc which cut the first arch.

4. Place the ruler on start point and where to arc intersect each other.

5. Draw 60 degree line.
. Watch the four drawing steps of 60 degree angle.

. Draw a new arc from Intersection of line and arc slightly longer than distance of arch.
. Draw another same size arch from Intersection of two archs.

. Place the ruler on start point and where to arc intersect each other.

. Draw 30 degree line.

Construction of a 30 Degree Angle
. Construct a perpendicular line.

. Place compass on intersection point.
. Adjust compass width to reach start point.
. Draw an arc that intersects perpendicular line.
. Place ruler on start point and where arc intersects perpendicular line.

. Draw 45 degree line.

Construction of a 45 Degree Angle
Construction of Same (Congruent) Angle

1. Draw an arc from the vertex of the angle.
2. Place compass to the intersection of line and arc.
3. Draw a new circle from this point to the other point.
4. Draw same circle to the other line to draw congruent angle.
5. Place ruler from the starting point of the line this intersection to draw congruent angle.
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